Hyperelliptic Curve Arithmetic

(A Shifty Affair)

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Joint work with **Michael John Jacobson, Jr.** and **Monireh Rezai Rad**, U Calgary

West Coast Number Theory
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Why Hyperelliptic Curves?

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We like even degree models!

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Hyperelliptic curve of genus g over $\mathbb{F}_q$ (q odd):

$$y^2 + h(x)y = f(x)$$

with

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For $g = 1$, the odd degree model yields the Weierstraß equation of an elliptic curve.

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Research into efficient arithmetic on even degree models is far less advanced.

Points, Divisors, Jacobian

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- AKA *degree zero divisor class group*;
- Similar to the (Arakelov) class group of a quadratic number field;
- For elliptic curves, this is just the group of points over $\mathbb{F}_q$.

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$$\mathcal{C} = [R - \deg(R)\infty]$$

- $\deg(R) \leq g$, with equality almost always (generic case)
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Explicit addition in $\mathrm{Jac}_{\mathbb{F}_q}(C)$: if R_1 represents $\mathcal{C}_1$ and R_2 represents $\mathcal{C}_2$, then the reduced divisor representing $\mathcal{C}_1 + \mathcal{C}_2$ is efficiently computable via Cantor's algorithm ("add & reduce").

Jacobian Arithmetic, Even Degree Models

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- $-\lceil g/2 \rceil \leq n \leq \lfloor g/2 \rfloor - \deg(R)$ gets rid of the extra adjustment steps — balanced representation (Galbraith-Harrison-Mireles Morales 2008)

Generic situation

For generic divisors ($\deg(R) = g$):

Paulus-Rück: $D = R - g\infty^+$

Balanced representation: $D = R - \left\lfloor \frac{g}{2} \right\rfloor \infty^- - \left\lceil \frac{g}{2} \right\rceil \infty^+$

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Cantor's Algorithm:

- *Divisor addition* — yields a finite divisor of degree $2g$
- *Divisor reduction* — $\lceil g/2 \rceil$ steps:
  - $\lfloor g/2 \rfloor$ subtractions by $\infty^+ - \infty_-$
  - For g odd, one subtraction by ∞^-

Paulus-Rück Arithmetic

For simplicity, assume that g is even. Consider two generic divisors

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Need $g/2$ additions by $\infty^+ - \infty^-$ to obtain the unique Paulus-Rück representative in the class of $D_1 + D_2$.

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In general, $\lfloor g/2 \rfloor$ extra steps are required.

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For g odd, one additional *balancing* step is needed.

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Mireles Morales declared this speed disadvantage in essence the final nail in the infrastructure arithmetic coffin (he was likely unaware of JSS 2007).

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- Final nail in infrastructure arithmetic coffin after all — but for a somewhat different reason

Conclusion

We implemented arithmetic in three frameworks for hyperelliptic curves:

- Jacobian, odd degree models
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- Infrastructure with JSS 2007 improvements, even degree models

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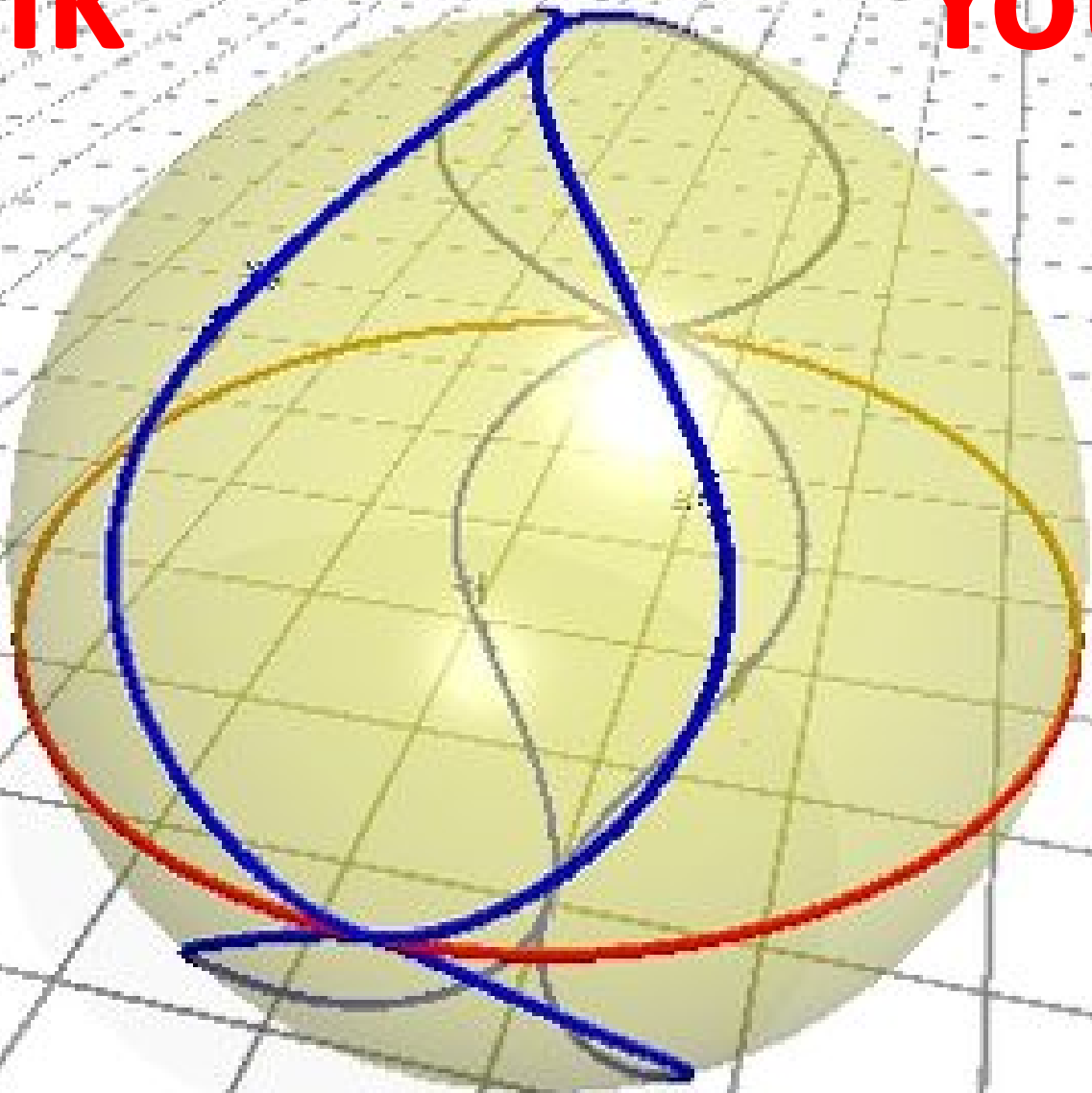
It may well be possible to improve the genus 3 formulas for even degree.

Thank You !



$$y^2=x^6+x^2+x$$

http://voltage.typepad.com/superconductor/2011/09/a-projective-imaginary-hyperelliptic-curve.html