Communications from a Higher Dimension: Recursive Triangles Embedded in Families of 1-Dimensional Recursive Sequences

Russell Jay Hendel
Towson University
RHendel@Towson.Edu

Jubillee WCNT, Asilomar

Dec 16-20, 2019

3:10-3:25, Dec 17, 2019

Simple Example: *Recursion*

- Recursion: $G_n=G_{n-3}-G_{n-2}-G_{n-1}$, $R=<1,-1,-1>$
- Initial Values: $G_0=1, G_1=0, G_2=0$, $IV=10^2$
- Further values: $G_3=1(1)-1(0)-1(0)=R.IV = 1$
- Further Values: $G_4 = \quad 1(0)-1(0)-1(1)=-1$
- Vector Not: $R.10^2=1$, $R.0^21=-1$, $R.<0,1,-1>=0$
- Obtain sequence: 1,0,0,1,-1,0,2,-3,1,4,-8,….
-

Simple Example: *Family*

- $G_{n-3}-G_{n-2}-G_{n-1}$ $<1,-1,-1>$, 10^2,0,1-,1,0,2…
- $G_{n-4}-\sum_{1\le i\le 3} G_{n-i}$: 10^3;0,1,-1,0,0,2,-3,1,0,4,-8
- $G_{n-5}-\sum_{1\le i\le 4} G_{n-i}$: 10^4;0,1,-1,0,0,0,2,-3,1,0,0,4,
- R=$<1,-1^{k-1}>$,IV=10^{k-1};0,1,-1,0^{k-2},2,-3,1,0^{k-3}..
- Row 3 Row 4 ….
- IDEA: 0s $\leftarrow\rightarrow$ Boundaries, non 0s$\leftarrow\rightarrow$ Rows
- Rows of length 2,3,…Naturally form a triangle

The Trianngle

- 1, -1
- 2, -3, 1
- 4, -8, 5, -1,
- 8,-20,18,-7,1
- Unexpected: Recursive Triangle
- $T_{r,c}=-T_{r-1,c-1}+2T_{r-1,c}$: $T_{r,1}=2^{r-3}$; $T_{r,r-1}=(-1)^r$

Research Questions

- Which families (of recursive seqeuences) produce recursive triangles
- Recursive Relations: 1D Recursive seq; Triangle recursion
- NOTE: You can take R=<1,-1^k> and perturb
- But if coefficiennts ||R||>>1→s. blows up→no 0s→No boundaries → No rows → No Triangles
- Proof methods (Nested inductions)

Theorem

- Recursive Family: $<1,-1^k,-m,-1^p>$
- k,m constants, p parameter → oo
- IV: 10^{k+1+p}
- Then
- There is a resultant triangle (←→0s, boundaries,rows) embedded in family
- Triangle governed by following recursion
- $<-1,2,0^{k-1},m-1,-(m-1)>$

Theorem

- Recursive Family: $<1,-1^k,m,-1^p>$
- k,m constants, p parameter → oo
- IV: 10^{k+1+p}
- Then
- There is a resultant triangle (←→0s, boundaries,rows) embedded in family
- Triangle governed by following recursion
- $<-1,2,0^{k-1},m-1,-(m-1)>$

Theorem Illustration

- Recursive Family: $<1,-1^k,m,-1^p>$
- K=5, M = 8

1	-1							
-7	*14*	*-7*	*0*	*0*	*0*	*2*	*-3*	*1*
49	-147	147	-49	0	0	-28	70	*-56*
-343	1372	-2058	1372	-343	0	294	-1029	1323

- -56 = -14+2*(-7) +0…+0 +(8-1)*(-3-1)
- $<-1,2,0^{k-1},m-1,-(m-1)>$

Proofs

- Base case:
  - You have to "prove" 1st Row: 1,-1, zeroes
  - Then you have to "prove" 2nd row
    - Proof is for all family members
  - Two rows: You can check triangle recursion
- Induction step
- Proof skillfully uses word←→vector notation
- Reduces computationality of recursive proofs
- We will illustrate: Prove 1st row

Proof: 1st Row Proper (illustration)

- Notation: $G_0=1, G_1=0, G_2=0$ ←→ 10^2
- Instead of speaking about G_3, use 10^2, G_{next}
- Need to prove:
  - $G_{next}=1, G_{next+1}=-1$ ← 1st Row
  - $G_{next+a}= 0$ $(2 \leq a \leq k)$ ← 1st Row border
- $G_{next}=1$ ← Apply G_{n-3}-G_{n-2}-G_{n-1} to 10^2
- Key point: Plugging in "=" Dot product
- $R.10^2 =$ <1,-1,-1> <1,0,0> $= 1$
- G_{n-3}-G_{n-2} … G_0 G_1 G_2

Proof: 1st Row +Boundaries

- R: $<1,-1^k>$ IV: 10^k Prop: $G_{next+0:k} = <1,-1,0^{k-1}>$
- $G_{next} = <1,-1^k>. 10^k = 1$ (1,0,0,…0→ 1)
- $G_{next+1} = <1,-1^k> <0^k,1> =-1$
- $G_{next+2} = <1,-1^k> <0^{k-1},1,-1> =0$
- $G_{next+3} = <1,-1^k> <0^{k-2},1,-1> =0$
- $G_{next+a} = <1,-1^k> <0^{k-(a-1)},1,-1> =0$ provided
  - First component in rh vector =0 (so that 1+-1 = 0>
  - k-(a-1) >=1
  - k >= a-1+1 =a Q.E.D.
-